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ABSTRACT

A discussion of the papers, "Representation, Vision and Visualization: Cognitive Functions in Mathematical Thinking. Basic Issues for Learning" (Raymond Duval) and "On the Development of Human Representational Competence from an Evolutionary Point of View: From Episodic to Virtual Culture" (James J. Kaput), is presented. Kaput points to the accelerating emergence of increasingly virtual worlds with which and through which humans interact. Duval emphasizes that thoughtful, didactic attention must be given to helping students employ any register of representation powerfully and flexibly, and that deep mathematics emerges from coordinating specific register-centered activities. This paper suggests that it would be interesting for Duval to analyze Kaput's virtual worlds in terms of registers of representations and didactical strategies, and for Kaput to analyze Duval's didactics in terms of virtual culture. (Contains 10 references.) (DDR)

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REPRESENTATION AND EVOLUTION: A DISCUSSION OF DUVAL'S AND KAPUT'S PAPERS[1]

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Duval and Kaput present two very differently-oriented perspectives on the important issues of representation in mathematics education. Yet, without setting out to do so, each paper speaks directly to issues raised by the other. I shall structure my comments by first focusing on the two papers independently and then on the two together.

Comments on Duval

Duval opens his paper with a comment that I found refreshing because it is so true:

> Research in developmental psychology, new technologies, new requirements in assessment have supported [needed changes over the past 50 years]. But their impact has been more effective on mathematics curriculum and on means of teaching than on the explanations of the deep processes of understanding and learning in mathematics.

Such explanations require explanatory frameworks, systems of constructs from which a researcher can formulate descriptions and explanations of important phenomena. Duval focuses on issues he sees as foundational to our understanding what conditions are propitious for mathematical learning. In the process he touches upon a myriad of distinctions that attempt to clarify essential ideas underlying representation and visualization. Among these are

- We never deal with mathematical objects, but only with representations of them
- "Representation," as commonly used, is ambiguous – that there is a common confounding of issues in thinking of "internal" versus "external" representations
- Representational activity is fundamentally semiotic in nature, and that semiotic systems are never transparent and must be developed within themselves

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- Semiotic activity occurs within registers of representing – systems of semiosis.
- Mature mathematical processing is founded on coordinating processes across semiotic systems.
- Mathematical visualization is different from perceiving, "to look at [drawings] is not enough to see, that is to notice and understand what is really represented," and is based on "operative apprehension" – seeing a present figuration as being but one possible state of a system of transformations.

Out of this Duval identifies three conditions for learning mathematics:

> to compare similar representations within the same register in order to discriminate what are the relevant values within a mathematical understanding, to convert a representation from one register to another one, and to discriminate the specific way of working in order to understand the mathematical processes which are perform in this register.

And he finally concludes with a statement directed at mathematics education researchers:

> We are here in front of an important field of research. But it seems still often neglected because most didactival studies are mainly centred on one side of the mathematical activity, as though mathematcial processes were natural and cognitively transparent.

Duval's emphasis on "registers of representation" (words, symbolic expressions, graphs, diagrams) reminds me of Post, Behr, Lesh, and Harel's ideas regarding modes of representation in their Rational Number Project investigations (Behr, Khoury, Harel, Post, & Lesh, 1997; Behr & Post, 1980; Behr, Harel, Post, & Lesh, 1993; Lesh, Behr, & Post, 1987). But it is different, too. The RNP's attention was on external figurations and meanings they possessed, whereas Duval's notion of representation (semiotic system) is more attuned to the activity of the representer. But his point is well taken that we must give explicit attention in instructional design to students' coordinating representational processes across registers. I am unsure, though, what Duval has in mind that is different from what Kaput (1987a; 1987b; 1989; 1992) has described as translating among representation systems and

working within a representation system. Kaput's definition of representation system is broad enough that it fits Duval's idea of a register, so it cannot be that Kaput talked about just (what might be called) symbol processing.

On the other hand, I wonder what, precisely, Duval means by a register, what he calls a system of representing. Is this an ad hoc construct, suggested to us by observing that there seem to be different but loosely equivalent ways of representing what appears to be a single idea? Or is it defined operationally by specifying cognitive operations that cohere into schemes that express themselves in equivalence classes of externalizations? That is, does Duval arrive at specific registers by identifying certain cognitive operations that express themselves in different settings in apparently different ways (thus, determining, from the individuals' perspectives, equivalent representations)? If so, the register is the scheme of operations. Otherwise, I don't know what a register is except that it is determined by social convention.

I also wondered whether Duval's appeal to semiotics was in the Sausserian or Peircian traditions. At times it is reminiscent of both – his reference to dyadic relationships being more Sausserian and his reference to triadic relationships being more Peircian. But it seems that Duval addressed a very different matter than either Saussere or Peirce. Saussere focused on semiotics without appeal to an external reality (whence dyadic relations between signifier and significant), whereas Peirce held a slot for an objective referent. But both Peirce and Saussere imagined an active interpreter who made a signifier into a sign. However, Duval agrees very much with Saussere and Peirce in the importance of talking about *people* developing and coordinating semiotic systems. As Chandler (1999) notes,

> This highlights the *process* of semiosis (which is very much a Peircean concept). The meaning of a sign is not contained within it, but arises in its interpretation. Whether a dyadic or triadic model is adopted, the role of the interpreter must be accounted for - either within the formal model of the sign, or as an essential part of the process of semiosis.

Comments on Kaput

Kaput places issues of representation into a larger perspective of evolutionary psychology. As I am unfamiliar with Donald's book, I shall take Kaput's fascinating account as being an acceptable presentation of it. He recaps Donald's (1991) theory that three major advances in human culture occurred in consonance with fundamental changes in human cognition.

Socialization emerged with the emergence of episodic memory, foundations of semiotic man emerged with the emergence of the capacity to use one item of experience to refer to another. Historical and persisting cultures emerged by way of humans' capability to experience events vicariously through telling and listening to stories. Theoretic culture emerged as a byproduct of humans' capacity to reason formally about their actual use of semiotic items – to attend to matters of form in their use of signs and symbols.

Kaput extrapolates from Donald's theory to suggest that human culture is at the dawn of yet another stage, a stage that is enabled by human's capacity to produce autonomous computations. This is the stage of virtual culture, brought about by informational interconnectivity on a massive scale.

While I am fascinated by Kaput's ideas, I wonder if he has changed Donald's thesis in subtle, fundamental ways. His presentation of evolution has, at times, a decidedly Lamarckian and teleological flavor.

Modern genetics differentiates between genotype and phenotype. As I understand it, a genotype has to do with the genetic structure inherited across generations, whereas phenotype is the set of characteristics exhibited by members sharing a common genotype. It is a tenet of modern genetics (I am told by my science education colleagues) that phenotype cannot influence genotype. Put simply, children of weight lifters will not inherit the fruits of their parents' efforts. They must exercise, too, in order that their bodies show the same characteristics as their parents' bodies. Now, the children of weight lifters may have a higher percentage of weight lifters among them than the general populace, but that is because they are around people who lift weights, not because of an inherited trait. Lamarckian biology, as I understand it, proposes that the phenotype can, in fact, influence the genotype.[1] This is not widely accepted, I am told, and is at best controversial.

Teleology is the idea that nature evolves in a way to reach a particular end. This, too, is rejected in modern genetics. That is, it is considered a mistake to make claims like "Frogs developed webbed feet so that they could swim," and like "Birds developed wings so that they could fly." Rather, more appropriate claims would be "Frogs that had webbed feet swam faster and with greater agility than frogs that didn't, and therefore had a higher survival rate in areas where large fish also populated the waters." They did not develop webbed feet in order to escape from fish. Instead, those who *had inherited that mutation* ended up with higher escape rates. Not all

[1] Piaget subscribed to this view based on research he conducted in his youth on the shells of fresh water mollusks. Smooth-shelled mollusks moved from placid waters to fast-flowing waters developed ripples in their shells. Offsprings of these mollusks, placed in placid waters, had rippled shells as did their offsprings.

mutations make a difference. Some make a positive difference, some make a negative difference, and some make no difference in survival rates.

One way to test for Lamarckianism in our understanding of culture is to imagine an infant transported from its native culture to an adoptive family in another. Infant Papuans brought to the United States to live with an upper middle-class family will probably exhibit all the characteristics of someone born to that culture (except perhaps for characteristics due to interactions with others that express others' attitudes toward children adopted from another culture).

Kaput's extension of Donald's theory seems to break with Donald's Darwinism. For example, Donald (as in a quotation presented by Kaput) made it clear that he did not think of language as an evolutionary breakthrough. Rather, he considered integrative thought as the evolutionary breakthrough. Language was an expression of this new genetic mutation in the face of pressures of persisting. Now, I say "persistence" instead of "survival" for a reason. Survival has *existence* at its core. To survive means continue to exist as a living entity. Persistence has *coherence* at its core. Persistence entails survival, but it also entails the pressures of abduction, reflection, and socialization (coordinating competing perspectives).

Kaput's extension does not point to any underlying change in the human genotype, and if there is such a change, he implies that it is because of current human activity – whence the Lamarckian flavor. Increased sunspot activity would obliterate the virtual culture overnight, but our cognitive potentialities would be unaffected. Kaput is probably correct that we are entering a new stage in human culture, but I do not see a deep connection with Donald's evolutionary psychology.

What Kaput and Duval Say to Each Other

Kaput pointed to the accelerating emergence of increasingly virtual worlds with which and through which humans interact. Duval emphasizes that thoughtful, didactic attention must be given to helping students employ any register of representation powerfully and flexibly, and that deep mathematics emerges from their coordinating across registers their specific register-centered activities. It would be interesting were Duval to analyze Kaput's virtual worlds for what he sees as registers of representations and didactical strategies to make them evident, and for Kaput to analyze Duval's didactics to see wherein it could be empowered by infusing it with perspectives of a virtual culture.

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